Invited Lecture

Modeling and Digital Technologies: Experiences and Challenges for Teacher Education

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ABSTRACT This lecture presents a didactic proposal that combines mathematical modeling and digital technologies in the framework of a teacher education program for future mathematics teachers in a public university of Argentina. After presenting the theoretical assumptions underpinning this proposal, the characteristics of the program and an annual mathematics education course that forms part of its curriculum are described. This course covers topics related to mathematical modeling and the use of digital technologies, among others. Details are given of the characteristics of the modeling scenario created within the framework of this course, for preservice teachers to experience the development of open modeling projects. A synthesis of the modeling experiences developed in 2020 during the COVID-19 pandemic is shown. These experiences were carried out in groups of preservice teachers, allowing them to choose freely a real-life topic of their own interest and the use of various digital technologies. The topics chosen by each group, the role of technologies, the learnings recognized by the preservice teachers and the difficulties and limitations detected are detailed. The text concludes with some reflections on the relevance of this type of experience in teacher education.

Keywords: Preservice teachers; Modeling scenario; Digital technologies.

1. Introduction

Curricular designs for secondary education in various countries and educational contexts present recommendations on the incorporation and use of digital technologies and modeling tasks for the teaching and learning of mathematics. However, the implementation of such recommendations is still scarce in several countries. In particular, this situation prevails in a significant number of regions throughout Argentina. While there are factors related to the lack of technological infrastructure, or the prevalence of a certain conservative academic culture that affects the acceptance of such recommendations, the literature reports that teacher education plays a key role in whether or not the incorporation of technologies (Clark-Wilson et al., 2014) and modeling (Blum, 2015; Doerr, 2007) at secondary school level is encouraged. In

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particular, it seems that if preservice teachers (PSTs) are given the opportunity to experiment with mathematical modeling tasks and technologies during their undergraduate education, it would contribute to making such tasks part of their teaching agendas in the future (Doerr, 2007; Lingefjärd, 2013; Villarreal and Esteley, 2023).

In the analysis of some of the official curriculum designs for the initial mathematics teacher education in Argentina (for example, Ministerio de Educación de la Provincia de Córdoba, 2010), the importance of the use of technology as a powerful auxiliary in the educational task is recognized. At the same time, mathematical modeling tasks are highlighted as a way of linking mathematics with the extra-mathematical real world. Meanwhile, the ways in which modeling and technologies are approached differ from one teacher education program to another. In this lecture, I present a particular pedagogical approach that involves modeling and digital technologies in a synergic way. This approach has been implemented, since 2010, in a mathematics teacher education program for preservice secondary school teachers at a public university from Córdoba (Argentina). The program lasts for 4 years and 66% of the syllabus is composed by mathematics courses, mainly taught by mathematicians, while the remaining 34% of the courses deal with educational issues, taught by pedagogues or mathematics educators. I have been working as a teacher educator in this program since 2006. For some years now, I have been in charge of an annual course called *Didáctica especial y taller de matemática*, which I will later describe and make reference to as the *Mathematics Education* (ME) course. Also, for other years of this period, I have been in charge of another annual course called *Metodología y práctica de la enseñanza* (*Teaching methodology and practice*). Within this course, the PSTs carry out their first teaching practices in secondary schools.

For the last fifteen years, these courses were scenarios in which we have been investigating, in conjunction with other colleagues from the university, different aspects related to: (1) PSTs executing mathematical modeling tasks (as modellers) and (2) PSTs implementing modeling tasks during their first teaching practices at secondary schools. In both cases, we observed that different technologies were important actors for enhancing the modeling processes, and then, the study of the impact of technologies on modeling performed by PSTs became one of the focuses of our research. For this lecture, I focus on PSTs' experiences while doing mathematical modeling (MM) with different types of digital technologies (DTs).

In what follows, after presenting some theoretical ideas on MM and DTs, I describe the modeling scenario we created in the framework of the ME course for PSTs to have MM experiences. Subsequently, I briefly report about topics that the PSTs selected, roles played by digital technologies, types of learning that occurred, difficulties that arose, and limitations that were recognized.

2. Pedagogical and Epistemological Assumptions regarding MM and DTs

In this section, I present the theoretical background that supports our actions as teacher educators and researchers. On one hand, there is the modeling perspective that we adopted in the teacher education program at the university. On the other hand, we have the epistemological perspective we hold concerning the use of DTs in the production of knowledge, and in the teaching and learning of mathematics.

The modeling perspective that we adopted is characterized by the following principles:

- Open nature of the activities posed to the students with no predetermined mathematical knowledge to be taught.
- Interdisciplinary nature of the work.
- Promotion of reflections about mathematics itself, the models created, and the social role of mathematics and mathematical modeling.
- Mastery of the whole modeling process considering all the phases of the cycle.

Given that there isn't any previewed mathematical content to be learned through the modeling projects, the focus is on modeling as a mathematical activity that deserves to be taught and learned for its own sake. Our didactical approach to the teaching of modeling is compatible with the perspective known as *modeling-as-content* (Julie and Mudaly, 2007), with the notion of *active modeling* proposed by Muller and Burkardt (2007), the *socio-critical perspective* of MM discussed by many Brazilian authors such as Araújo (2012, 2010, 2009), Barbosa (2006), Silva and Kato (2012), or the ideas of *project work* described by Ole Skovsmose, in the framework of critical mathematics education (Skovsmose, 1994, 2001).

Regarding the use of technology, we have adopted the idea that knowledge is produced by collectives of *humans-with-media* (Borba and Villarreal, 2005). The word media refers to any kind of tool, device, equipment, instrument, artefact, or material resulting from technological developments. For this lecture, I am interested specifically in DTs, which include the Internet, any kind of mathematical software, and programming languages.

The notion of *humans-with-media* was presented in Borba and Villarreal (2005), and it is associated with two main ideas. One is that cognition is not an individual enterprise, but a social one, which is why the construct explicitly includes humans, in plural. The other key idea is that cognition includes tools, media with which the knowledge is produced. This component of the epistemic subject is not auxiliary or complementary, but essential. Media are constitutive elements of knowledge. The integration of DTs reorganizes the thinking and production of knowledge. This reorganization may imply transformations in educational environments; for example, in the kind of problems that may be addressed, in the ways of solving them and in the manners of validating and communicating results. Borba and Villarreal (2005) also explored the synergic relationship between modeling and technology, and other authors, such as Greefrath (2011) and Doerr *et al*. (2017), among many others, refer to the role of technology in modeling. Some examples of strong influence of DTs on the development of open modeling projects in different educational contexts can be found in Borba *et al*. (2016), Villarreal *et al*. (2010), and Villarreal *et al*. (2018).

In the next section, I present the features of a *modeling scenario* we designed for our teacher education program and the reasons that justify such design. This scenario was also the focus of several investigations.

3. A Mathematical *Modeling Scenario* in our Local Context

In Argentina, the official curricular documents for secondary school and the national standards for teacher education programs recommend the introduction of MM and the use of DTs for the teaching and learning of mathematics.

These recommendations imply challenges for teacher education, in general, and particularly for preservice teacher education. Meanwhile, the reality of our university teacher education program is far removed from heeding such recommendations. On one hand, mathematical courses usually give little or no room for active modeling. On the other hand, although technologies are gaining terrain in our program, they usually have a supplementary role, with little use of their potential for enhancing mathematical thinking and learning.

To reverse this situation in our local context, in 2010, we decided to create a *mathematical modeling scenario* within the regular ME course. Since then, at least one of the researchers of our research group is in charge of the course.

The ME course is in the third year of the teacher education program; it extends for 30 weeks with two 4-hour classes per week. In this course, several trends in mathematics education are studied, for example, problem-solving, critical mathematics education, the use of technology in mathematics education, and mathematical modeling.

The notions of model, mathematical model and mathematical modeling process are discussed in the course. The phases of a MM process are studied using different modeling cycles such as those proposed by Bassanezi or Blomhøj. Fig. 1 shows a modeling cycle adapted from Bassanezi (2002) and Fig. 2 shows the one proposed in Blomhøj (2004).

During the classes, experiences of modeling activities in different educational contexts are analyzed, and several modeling tasks are solved. Finally, the PSTs are invited to develop their modeling projects freely using DTs, if they wish. For this, the PSTs are asked to form small groups and select a (non-mathematical) real-world theme of their interest, formulate problems related to this theme, select variables, raise hypotheses, design experiments (if necessary), search for information, collect and process data, and solve the problems. Each group has to write a report and make an oral presentation for the whole class. During such presentations, which last about 40 minutes, the rest of the class ask questions and make comments about the projects. In many cases, the students discuss possible modeling tasks for the secondary school, or they reflect on the role of technology in the modeling process. Further on, I will refer to the projects carried out in 2020 and present a detailed timeline with the activities that guided the development of the modeling projects of that year.

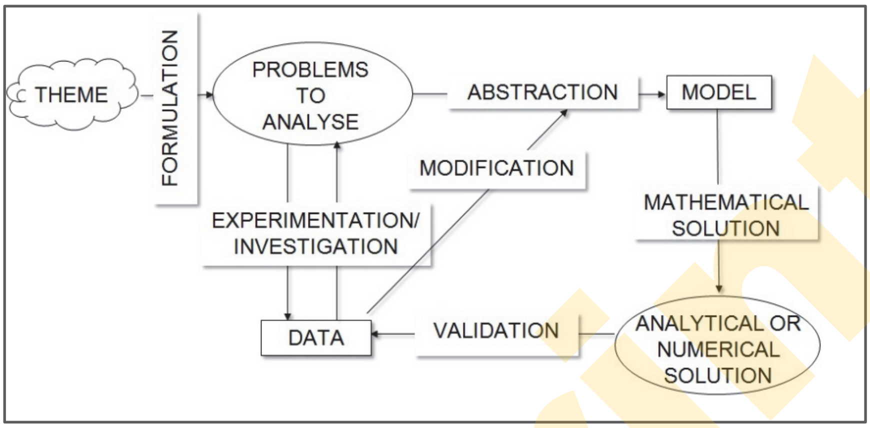

Fig. 1. Modeling cycle adapted with permission from Bassanezi (2002, p. 27)

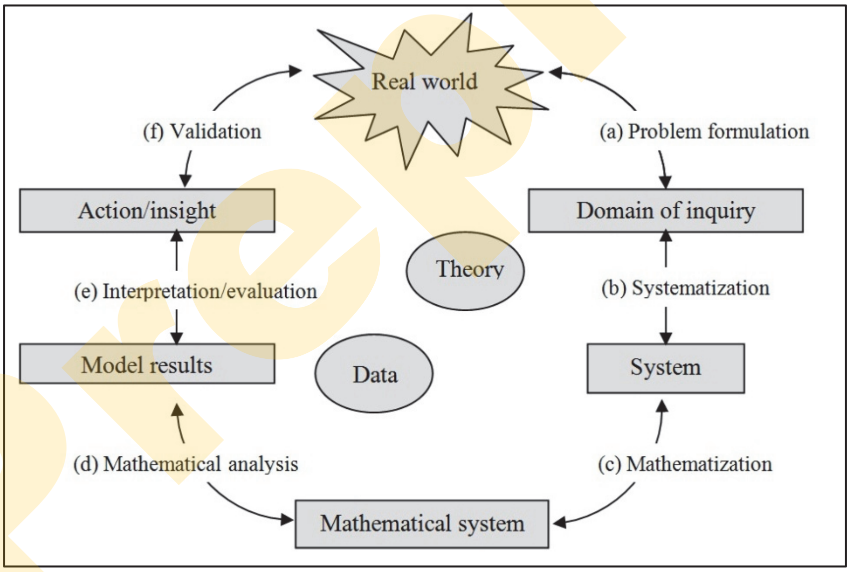

Fig. 2. Modeling cycle proposed in Blomhøj (2004, p. 148)

This scenario offers the PSTs the opportunity of experiencing a complete open modeling process, following the phases of the modeling cycle. Such experiences have been registered in different ways: PTS' final written reports, our field notes during the classes, GeoGebra files, spreadsheets files, Python codes, and videotapes of the final oral presentations. These are the sources that allowed us to develop different studies in the period between 2010-2020. So the MM scenario became a research scenario, in which we analyzed, for example, the themes that the PSTs selected for their modeling projects, the reasons for such selection, and the relation with social concerns. We also analyzed the mathematical contents that the PSTs used in their projects. The results of this analysis were published in Villarreal *et al*. (2015) and Villarreal (2019).

When focusing on the integration of DTs in modeling processes, we sought to determine which technologies PSTs chose, for which modeling purposes, and in which phases of the modeling process the technologies were significantly used. These questions were addressed in Villarreal *et al*. (2018). In this case, it was found that PSTs used the Internet, spreadsheets, mathematical software and programming languages. The Internet was the most utilized technology. It was used to find information or data, to select variables, or to formulate problems. The other three technologies significantly influenced the processes of mathematical solution and validation.

The next sections describe the experience in 2020, during the pandemic. First, I will describe, in general, the organization of the ME course in distance mode, followed by the work with modeling projects, in particular. Then, I will present brief descriptions of the MM projects that four groups of PSTs carried out.

4. Mathematical Modeling during the Pandemic in 2020

4.1. The organization of the ME course in distance mode

In 2020 the ME course was held in a distance learning mode, except for the first two classes. Thirteen students attended the course and three teachers were in charge of it. The course started in the week of March 16th, and on Friday of that week, mandatory quarantine was decreed all over the country. Classes at all levels of education moved to an online modality.

In the case of the ME course, for some years now, we have used a virtual classroom on the Moodle platform (Fig. 3, see in the next page) in which we upload texts, activities, videos, PowerPoint presentations of the classes, among other materials. In the transition to distance learning, this tool was fundamental, but we realized that this resource alone was not enough. So, a few weeks after starting the quarantine, we decided to start virtual meetings with the students twice a week through Google Meet.

This brought on other issues. Many students did not have good connectivity at home, some did not have a computer with a camera and others could only connect through their mobile phones, with multiple interruptions and with expensive costs for internet connection. There were students whom we rarely saw, given they could not turn on their cameras due to poor connectivity and, at times, even the teachers had connectivity problems. One student and one of the teachers had to manage childcare while in class.

As demonstrated in other levels of the educational system in Argentina, the pandemic revealed the unequal conditions for university students to access education in this public health emergency.

Fig. 3. Snapshot of the ME virtual classroom in Moodle.

4.2. The organization of work with MM projects

The thirteen students in the course were divided into four groups: one group of four members and three groups of three members. Fig. 4 shows a timeline with dates and activities related to the development of the MM projects. It shows the events from the first class in which MM was introduced as a trend in mathematics education, to the submission of the final written report of each group.

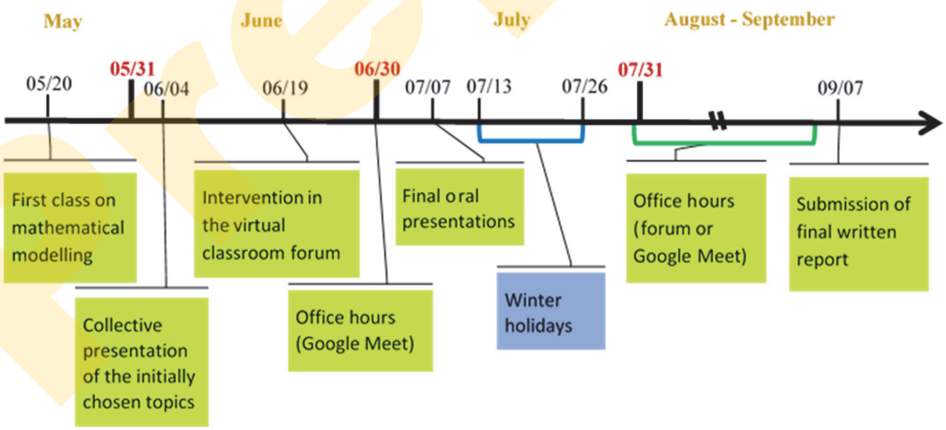

Fig. 4. Timeline of the development of MM projects in 2020

The moment of the collective presentation of the first topics chosen by the groups was fundamental since, based on the exchange with classmates and teachers, each group finally decided which topic to address. After this initial exchange, the activities related to the modeling projects were carried out by the PSTs in an autonomous way in schedules outside the classes. A forum was created in the Moodle virtual classroom and a first intervention on the progress of the projects was requested for June 19th. Each group could add a new discussion topic, whenever needed, to send questions and receive advice from the teachers. Previous to the final oral presentation, office hours were arranged with each group through Google Meet.

The teachers acted as guides that helped to formulate or reformulate the problems, inform about possible sources of data, and suggest new questions to make the students engage in more complex modeling processes.

The final oral presentations of the groups allowed us to have an overview of the state of progress of each project. After the oral presentations, the PSTs had 2 months to complete the written report, considering all the suggestions and observations received during the oral presentations. During this 2-month period, the PSTs also consulted with the teachers to make progress in their writing. They did it via the forum or Google Meet.

In the next section I will report on the four MM projects that PSTs developed in 2020.

5. The Four PMTs' Modeling Projects

To give a comprehensive overview of the four modeling projects, in this section I will briefly report about: the topics that the PSTs selected, the roles played by digital technologies, the types of learning that occurred, and the difficulties and limitations that were recognized.

5.1. The selected topics

I will refer to the topics that each group initially chose and which topic was finally addressed in each one.

Group 1 thought about four possible themes: (1) Lottery games, (2), Canes for the blind (in this group, one of the students is blind), (3) Body mass index, and (4) COVID-19.

After the exchange with teachers and colleagues, and considering the large amount of data available on COVID-19, the group finally chose this topic. The first questions were broad and difficult to address in the available time:

- Taking into account the reliable data, is it possible to predict the evolution of this disease in Argentina?
- Can this possible prediction be compared, or not, with a situation in which we have not taken health prevention measures?
- Is it possible to make a calculation that would allow us to estimate, more or less quickly, whether we are better off, worse off or in the same situation in relation to the evolution of the disease?

These questions were very ambitious and with the advice of the teachers, the group decided to consider data only from our province (Córdoba) and to make a descriptive model of the situation, analyzing the effects of the restriction or relaxation measures in the period between March and July 2020.

Group 2 thought about the following issues: (1) Evolution of prices to build a house from 2015 to 2029, (2) Causes of undergraduate students dropping out of the FAMAF (the faculty in which they are studying), (3) Electronic waste in Córdoba, and (4) Femicides in Argentina.

According to the members of the group, the topic of femicides "was a more current one for us to choose, with elements of analysis that can be enriching when it comes to modeling". They also stressed the importance of sharing the results of their research, as an important way to raise awareness of the severity of this social problem.

As in the case of Group 1, the data available from different official sources for the period 2014-2019 influenced the formulation of the questions. The following are some of these questions:

- Is there a trend in the number of femicides in Córdoba from 2014 to 2019?
- In which province of Argentina was the highest number of victims registered in the period between 2014-2019?
- What was the distribution of the femicide rate in Argentina in 2019?

Group 3 proposed to address a topic related to technology and health. The students were concerned with the amount of time a person spends in front of a screen, taking into account the situation of preventive and mandatory social isolation due to the COVID-19 pandemic. The group posed the following question: How can screen time affect a person's health during this quarantine time? To answer this question, new questions were posed:

- How much time does a person spend in front of a screen per day, per week, during the quarantine period?
- Is screen time the same on a weekday as on a weekend day?
- For what purposes are the devices used?
- What effects can too many hours in front of a screen cause, and do all devices have the same effect?

Group 4 thought about the following themes: (1) The impact of the quarantine on access to education, (2) Gender-based violence in Argentina due to the pandemic, (3) Decrease of contamination after compulsory isolation, and (4) Potato production in Argentina.

The group finally chose the first topic, but a reformulation was necessary since it was a very broad subject, with many aspects to explore. The final decision was conditioned by the possibility of accessing real data from the responses to a survey posted on the Internet, about computer accessibility conditions among first and second-year students of the faculty. This survey had been carried out by the Student Centre to learn the technological needs of undergraduate students to be able to continue their studies at the university. Finally, the topic chosen was: Internet access in first and second-year students of FAMAF during the pandemic. From this topic, the group formulated the following question: How much does the lack of internet access affect the dropout rate of first and second-year FAMAF students during the pandemic?

In all cases, the final decision on the topic to be addressed and the questions posed were significantly influenced by the pandemic and confinement situation we were experiencing, and the consequent profusion of data associated with this situation that were available for use. In addition, the first discussions with the whole class at the time of the collective presentations also played a decisive role in the final decision on the problems to be addressed in each project.

5.2. Roles played by digital technologies

Regarding the technologies used in the MM projects, the most popular were: spreadsheets, GeoGebra and the Internet. Spreadsheets were used to systematize data in tables, make pie or bar graphs to represent data, and count cells using the Excel Filter function and the COUNTIF function to analyze answers in a survey. GeoGebra was used to fit curves to a dataset and represent data in a coordinate system. The Internet was used to search for information on the topics to be studied (e.g. effects of prolonged screen time), watch lectures on different mathematical models associated with the spread of the coronavirus, watch tutorials on how to use Excel functions, have access to official agency websites to collect data on the number of people infected with coronavirus per day in the province of Córdoba, or the number of femicides in Argentina in a given period.

As we had observed in previous years (Villarreal *et al*., 2018), the use of DTs was significant in the phases of problem formulation, abstraction and mathematical resolution (see phases in Fig. 1).

5.3. Types of learning that occurred according to the PSTs

When the PSTs were asked what they had learned, all the groups reported specific learning related to the selected topics: awareness of the complexity of the health situation caused by COVID-19, the different issues associated with violence against women and femicides, awareness of the negative effects of excessive screen time on eyesight and posture and recommendations to mitigate these effects.

PSTs also highlighted their learning around the use of some spreadsheet functions and GeoGebra commands to fit curves.

Regarding mathematics and its use, PSTs stated that, in the development of their modeling projects, mathematics was a tool to read reality. Sometimes the mathematics used was simple, but it had a clear application. Every index, every rate that was calculated had a meaning. A number could represent a dead woman, a sick person, a student who couldn't access education or the indicator that we need to stop spending so much time in front of a screen.

PSTs also acknowledged that there was learning related to statistical work. For example, the elaboration of summary tables from a dataset, the selection of variables, the creation of categories, the construction of contingency tables to analyze the existence, or not, of relationships between variables, and the interpretation of tables.

In summary, PSTs identified learning in four areas: in relation to the topics addressed, in relation to the use of DTs, in relation to mathematics itself, and in relation to the use of mathematics for critical reading and understanding of the world. These learnings are evidence of a socio-critical modeling perspective in action.

5.4. Difficulties and limitations

One of the major difficulties reported by the PSTs during the modeling process was the impossibility of meeting face-to-face to work. This made it difficult to agree on actions and make decisions smoothly. One group decided to break the quarantine and meet to move forward.

Another difficulty was agreeing on when to have remote meetings through video calls. This was complicated due to the different responsibilities and activities of each student, and due to the poor internet connectivity of some of them.

The impossibility of meeting face-to-face involved situations such as that experienced by the members of the group working with COVID-19 (Group 1), which concerns the use of Excel. They had constructed a table recording the number of new coronavirus infections per day in Córdoba. They also needed to calculate the cumulative number of infections per day as shown in Fig. 5. Doing this calculation using Excel is trivial, but none of the students in the group knew how to do it. One of them tried to make a code using Python, to calculate the sum of the cumulative cases per day, but Python reported that there was an error in his code. This student is blind and was alone. As he could not meet with the colleagues of the group, it was difficult for him to detect the error in the code, because he could not see it. Therefore, they decided to do the calculations by hand, which involved a lot of manual work and a lot of time wasted. The students stated that they were not good users of Excel.

FECHA.	NUEVOS INFECTADOS.	acumulad
2-mar	1	1
3-mar	1	2
4-mar	0	2
5-mar	1	3
6-mar	1	4
7-mar	0	4
8-mar	1	5
9-mar	1	6
10-mar	0	6
11-mar	1	7
12-mar	2	9
13-mar	5	14
14-mar	4	18
15-mar	3	21
16-mar	8	29
17-mar	9	38
18-mar	5	43
19-mar	13	56

Fig. 5. Number of new infected persons per day and cumulative number in Córdoba city, between March 2 and March 19, 2020. Source: students' written report.

It seems that both not being able to meet face-to-face and having poor connectivity were major obstacles to the development of the MM projects.

In addition to the difficulties recognized by the PSTs, we (teachers in charge of the ME course) recognized some deficiencies in the teacher education program. The PSTs do not study differential equations and the study of descriptive statistics is very limited. This implies limitations when working with population growth problems or when statistical analysis of data was needed.

6. Some Final Remarks

Authors such as Doerr (2007), Niss *et al*. (2007), Blum (2015) and Gastón and Lawrence (2015) refer to the necessity of providing future teachers with opportunities of experiencing the modeling process during their preservice education to place modeling on the agenda of their future teaching activities. In line with these authors, Lingefjärd (2007, 2013) asserts that technology may broaden and enhance PSTs' experiences with modeling processes.

While what I have presented here is merely a description of the MM experiences of four groups of PSTs, it is an example of the kind of work that can be done in a teacher education program around MM in conjunction with DTs.

The modeling projects carried out by the PSTs and their reflections, expressed during oral presentations or written in the final report, provide evidence of the variety of topics addressed and their close link to the pandemic situation, the technologies used, the learning achievements, the difficulties encountered and the limitations recognized. In particular, some students made sense of the experience envisioning possible implications for their future as teachers.

In this lecture I have reported on the positive issues pointed out by the PSTs during the MM process carried out in pandemic times and about the difficulties they experienced. Despite the difficulties, and based on the positive evidence we gained through many years of experience, we argue that the implementation of open modeling activities and the use of technologies are imperative within preservice teacher education for many reasons:

- They can enhance learning at any level of the education system.
- They can contribute to mathematics being seen as a useful tool for describing and analyzing real problems, making informed decisions, and criticizing situations using solid arguments.
- They can make future teachers sensitive towards the different ways of making sense of mathematics.

Finally, it should be noted that while it is true that this type of experience is valuable for PSTs, the implementation of open MM tasks such as the ones described here, in the context of secondary education, implies an important challenge for teachers and students. In this sense, the study of the design and implementation of open MM tasks in secondary school classrooms has also been a topic of research in our research group. These studies permanently lead us to rethink the principles that characterize our modeling perspective and to reflect on different ways of working with MM and the use of DTs in mathematics teacher education.

Acknowledgments

This work was carried out with the financial support of the Secretaría de Ciencia y Técnica (UNC), the Agencia Nacional de Promoción de la Investigación, el Desarrollo Tecnológico y la Innovación, and the Consejo Nacional de Investigaciones Científicas y Técnicas.

References

J. L. Araújo (2012). Ser crítico em projetos de modelagem em uma perspectiva crítica de educação matemática. *Bolema*, 26, 839–859.

J. L.Araújo (2010). Brazilian research on modeling in mathematics education. *ZDM — Mathematics Education*, 42(3–4), 33–348.

J. L.Araújo (2009). Uma abordagem sócio-crítica da modelagem matemática: a perspectiva da educação matemática crítica. *Alexandria*, 2, 55–68.

J. C. Barbosa (2006). Mathematical modeling in classroom: A critical and discursive perspective. *ZDM — Mathematics Education*, 38(3), 293–301.

R. Bassanezi (2002). *Ensino-aprendizagem com modelagem matemática: uma nova estratégia*. São Paulo: Contexto.

M. Blomhøj (2004). Mathematical modeling — A theory for practice. In: B. Clarke, D. Clarke, G. Emanuelsson, B. Johnansson, D. Lambdin, F. Lester, A. Walby, and K. Walby (Eds.), *International Perspectives on Learning and Teaching Mathematics*. National Center for Mathematics Education, pp. 145–159.

W. Blum, (2015). Quality teaching of mathematical modeling: What do we know, what can we do? In: S. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education*. Cham: Springer, pp. 73–96.

M. Borba and M. Villarreal (2005). *Humans-With-Media and the Reorganization of Mathematical Thinking: Information and Communication Technologies, Modeling, Experimentation and Visualization*, *1st Ed*. New York, USA: Springer.

M. Borba, M. Villarreal, and D. Soares (2016). Modeling using data available on the internet. In: C. Hirsch and E. McDuffie (Eds.), *Annual Perspectives in Mathematics Education 2016: Mathematical Modeling and Modeling Mathematics*. NCTM, pp. 143–152.

A. Clark-Wilson, O. Robutti, and N. Sinclair (Eds.) (2014). *The Mathematics Teacher in the Digital Era. An International Perspective on Technology Focused Professional Development*. Springer.

H. Doerr (2007). What knowledge do teachers need for teaching mathematics through applications and modeling? In: W. Blum, P. Galbraith, H-W. Henn, and M. Niss (Eds.), *Modeling and Applications in Mathematics Education*. New York: Springer, pp. 69–78.

H. Doerr, J. Ärlebäck, and M. Misfeldt (2017). Representations of modeling in mathematics education. In: G. Stillman, W. Blum and G. Kaiser (Eds.), *Mathematical Modeling and Applications: Crossing and Researching Boundaries in Mathematics Education*. Springer, pp. 71–82.

J. Gastón and B. Lawrence (2015). Supporting teachers' learning about mathematical modeling. *Journal of Mathematics Research*, 7(4), 1–11.

G. Greefrath (2011). Using technologies: new possibilities of teaching and learning modeling — Overview. In: G. Kaiser, W. Blum, R. Borromeo Ferri, and G. Stillman (Eds.). *Trends in the Teaching and Learning of Mathematical Modeling*. New York: Springer, pp. 301–304.

C. Julie and V. Mudaly (2007). Mathematical modeling of social issues in school mathematics in South Africa. In: W. Blum, P. Galbraith, H. Henn, and M. Niss (Eds.), *Modeling and Applications in Mathematics Education — The 14th ICMI Study*. New York: Springer, pp. 503–510.

T. Lingefjärd (2007). Modeling in teacher education. In: W. Blum, P. Galbraith, H-W. Henn, and M. Niss (Eds.), *Modeling and Applications in Mathematics Education*. New York: Springer, pp. 475–482.

T. Lingefjärd (2013). Teaching mathematical modeling in teacher education: Efforts and results. In: X. -S. Yang (Ed.), *Mathematical Modeling with Multidisciplinary Applications*. Hoboken, NJ: John Wiley & Sons, Inc, pp. 57–80.

Ministerio de Educación de la Provincia de Córdoba (2010). *Diseño curricular de la Provincia de Córdoba. Profesorado de educación secundaria en Matemática*. Accessed 25 Feb 2022.

E. Muller and H. Burkhardt. Applications and modeling for mathematics. In: W. Blum, P. Galbraith, H. Henn, and M. Niss (Eds.). *Modeling and Applications in Mathematics education — The 14th ICMI Study*. New York, USA: Springer, 2007. p. 267–274.

M. Niss, W. Blum and P. Galbraith (2007). Introduction. In: W. Blum, P. Galbraith, H-W. Henn, and M. Niss (Eds.), *Modeling and Applications in Mathematics Education* (pp.3–32). New York: Springer.

C. D. Silva and L. A. Kato (2012). Quais elementos caracterizam uma atividade de modelagem matemática na perspectiva sociocrítica? *Bolema, Boletim de Educação Matemática*, 26 (43), 817–838.

O. Skovsmose (1994). *Towards A Philosophy of Critical Mathematics Education*. Kluwer.

O. Skovsmose (2001). Landscapes of investigations. *ZDM. Mathematics Education*, 33(4), 123–132.

M. Villarreal, C. Esteley and M. Mina (2010). Modeling empowered by information and communication technologies. *ZDM — Mathematics Education*, 42(3–4), 405–419.

M. Villarreal, C. Esteley and S. Smith (2015). Pre-service mathematics teachers' experiences in modeling projects from a socio-critical modeling perspective. In: G. Stillman, W. Blum and M. Biembengut (Eds.), *Mathematical Modeling in Education Research and Practice: Cultural, Social and Cognitive Influence*. Springer, pp. 567–578.

M. Villarreal, C. Esteley and S. Smith (2018). Pre-service teachers' experiences within modeling scenarios enriched by digital technologies. *ZDM — Mathematics Education*, 50(1–2), 327–341.

M. Villarreal (2019). Experiencias de modelización en la formación de futuros profesores de matemática. *Cuadernos de Investigación y Formación en Educación Matemática*, Year 14, N.18, 227–242.

M. Villarreal and C. Esteley (2003). Researching professional trajectories regarding the integration of digital technologies: The case of Vera, a novice mathematics teacher. In A. Clark-Wilson, O. Robutti, and N. Sinclair (Eds.), *The Mathematics Teacher in The Digital Era*. Springer, pp. 323−346.